

CEVAP ANAHTARI

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SORU1:

$X, Y \in \chi(E^3)$, $X = x_1^2 \frac{\partial}{\partial x_1} - x_1 x_3 \frac{\partial}{\partial x_3}$ ve $Y = x_1 \frac{\partial}{\partial x_1} - x_2 \frac{\partial}{\partial x_2} + x_1 x_2 \frac{\partial}{\partial x_3}$ veriliyor.

$P = (1, 2, 1) \in E^3$ için $D_X Y$ değeri aşağıdakilerden hangisidir?

- A) (1, 0, 2) B) (1, 1, 2) C) (1, 0, 0) D) (1, 1, -2) E) (1, 1, 0)

SORU2:

$v = (1, 0, 2) \in \mathbb{R}^3$, $f(x, y, z) = x^2 y + xyz$ ve $P = (1, -1, 2) \in E^3$ için $\vec{V}_P[f]$ değeri aşağıdakilerden hangisidir?

- A) -6 B) 6 C) -1 D) 1 E) 0

SORU3:

$X = (1, 2) \in \chi(E^2)$ vektör alanının $P = (-1, 2) \in E^2$ noktasından geçen maksimal integral eğrisi aşağıdakilerden hangisidir?

- A) $\alpha(t) = (t+1, 2t+2)$ B) $\alpha(t) = (t+1, 2t-2)$ C) $\alpha(t) = (t-1, 2t+2)$
D) $\alpha(t) = (t+1, -2t+2)$ E) $\alpha(t) = (t-1, 2t-2)$

SORU4:

$X = x_1 \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} = x_2^2 \frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_1}$ vektör alanının $\left\{ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3} \right\}$ bazına göre bileşenleri

aşağıdakilerden hangisidir?

- A) $(1, 1+x_1, x_2^2)$ B) $(1, -1, x_2^2)$ C) $(1-x_1, -1, -x_2^2)$ D) $(x_2^2, -1, 1+x_1)$
E) $(1+x_1, -1, x_2^2)$

$$1) D_x \gamma = (X[y_1], X[y_2], X[y_3])$$

$$X[y_1] = X[x_1] = x_1^2 \frac{\partial x_1}{\partial x_1} + 0 + (-x_1 x_3) \frac{\partial x_1}{\partial x_3} = x_1^2$$

$$X[y_2] = X[-x_2] = x_1^2 \frac{\partial (-x_2)}{\partial x_1} + 0 + (-x_1 x_3) \frac{\partial (-x_2)}{\partial x_3} = 0$$

$$X[y_3] = X[x_1 x_2] = x_1^2 \frac{\partial (x_1 x_2)}{\partial x_1} + 0 + (-x_1 x_3) \frac{\partial (x_1 x_2)}{\partial x_3} = x_1^2 x_2$$

$$\Rightarrow D_x \gamma = (x_1^2, 0, x_1^2 x_2)$$

$$\Rightarrow (D_x \gamma)(p) = (1, 0, 2) \quad \text{Cevap A ziklidir.}$$

$$2) \vec{\nabla}_p [f] = 1 \cdot \frac{\partial f}{\partial x} \Big|_p + 0 \frac{\partial f}{\partial y} \Big|_p + 2 \frac{\partial f}{\partial z} \Big|_p$$

$$= 1 \cdot (2xy + y^2) \Big|_p + 2(xy) \Big|_p$$

$$= 2p_1 p_2 + p_2 p_3 + 2p_1 p_2$$

$$= -2 - 2 - 2$$

$$= -6 \quad \text{Cevap A ziklidir}$$

$$3) X(\alpha(t)) = \alpha'(t)$$

$$\rightarrow (1, 2) = \left(\frac{d\alpha_1}{dt}, \frac{d\alpha_2}{dt} \right) \Rightarrow \alpha_1(t) = t + C_1, \quad \alpha_2(t) = 2t + C_2 \Rightarrow \alpha(t) = (t + C_1, 2t + C_2)$$

$$\alpha(0) = p \Rightarrow (C_1, C_2) = p = (-1, 2)$$

$$\Rightarrow C_1 = -1, C_2 = 2$$

$$\Rightarrow \alpha(t) = (t - 1, 2t + 2)$$

$$\text{Cevap C ziklidir.}$$

$$4) X = (1 + x_1) \frac{\partial}{\partial x_1} + (-1) \frac{\partial}{\partial x_2} + x_2^2 \frac{\partial}{\partial x_3}$$

$$\text{Cevap E ziklidir.}$$